behavior of these curves is the same as one generally finds in practice.

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Nonunique Solutions in Unsteady Transonic Flow

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Introduction

THE need to properly study the existence of nonunique solutions of the partial differential equations that govern transonic flowfields remains a problem in computational fluid dynamics. The need to determine whether these solutions are physically realizable, or are manifestations of the numerical modeling, is based on applications of these solutions. In particular, for unsteady flows, accurate dynamic load prediction is necessary for analysis and prediction of aeroelastic response.

Previously, several authors have commented on the causes of nonunique solutions in steady, small-disturbance, transonic flow models. Initially, Steinhoff and Jameson[1] pointed out the existence of nonuniqueness in steady, full potential flow solutions. They concluded from their steady-flow analysis, that the nonuniqueness appeared in a narrow band of freestream Mach numbers between 0.82–0.85. Since this Mach number band coincides with that in which buffet phenomena occur, a link was drawn between the presence of multiple solutions and real physical phenomena, and it was concluded that nonuniqueness was not merely a consequence of the numerical modeling.

An interesting comparison of the potential flow nonuniqueness phenomena with one-dimensional nozzle flow was suggested by Salas et al.[2] If the nozzle flow is choked, then what occurs in the divergent section is dependent on the downstream pressure. One possibility is a compression to subsonic flow. A second possibility is an isentropic compression at a supersonic speed. Otherwise, a shock wave could form in the divergent part of the nozzle with subsequent deceleration to subsonic flow. The authors suggest that the inability of the conservative potential equation to correctly position the shock wave, could be responsible for the nonuniqueness.

For unsteady flows, Williams, Bland, and Edwards[3] examined the nonuniqueness problem using a small-disturbance, potential flow model. The response of the NACA 00XX series of airfoils to a pitch pulse was examined. In this case, the airfoil undergoes a rapid increase in angle of attack, followed by a rapid decrease to the initial conditions. The flow responds to this sudden change, and then settles down in time, back to the initial steady flow conditions of zero lift. However, nonzero lift conditions were produced for the range of Mach numbers 0.835–0.858. They also noted that in this Mach number range, the shock waves formed between 72–88% chord.

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It is important to note that shock wave location is based on downstream pressure. The authors concluded, that because small-disturbance potential flow models and full potential flow models share the assumption of isentropic flow, the nonuniqueness would also occur in full potential flow models as well.

These same conclusions were noted by Fuglsang[4] who used the unsteady, transonic small disturbance code XTRAN2L[5] to carry out his computations. A NACA 0012 airfoil was subjected to a sudden change in angle of attack, at freestream Mach numbers between 0.82–0.85. The ensuing load distribution was determined as a function of time. As the pitch angle decayed in time, it was found that for a certain range of freestream Mach numbers, the lift coefficient did not decay to zero lift, thus indicating the presence of nonunique solutions.

The purpose of this work is to study the occurrence of nonunique solutions in unsteady transonic flows. In the present study, a full potential, unsteady code was developed and used. This code was applied to the analysis of flows past NACA 0012 and NACA 64A006 airfoils. Previous researchers[4,6] found nonunique solutions using transonic small disturbance (TSD) models. Our more accurate, full potential model, showed that these nonunique solutions do not actually exist in certain cases. These results are important in helping to determine the conditions under which nonunique solutions are possible in unsteady, computational flow models. To date, there has been no verification of nonunique solutions in unsteady flow, using either full potential or Euler codes. Details of the development and validation of the full potential code are given in Murty.[7]

Pitch Pulse Response

Fuglsang used XTRAN2L,[5] an unsteady small disturbance code, for all his computations. Starting from a steady-state solution, Fuglsang applied a pitch pulse

$$\alpha(t) = 0.00436 \exp[-0.0025(t - 100\Delta t)^2]$$

to the NACA 0012 airfoil. As the pitch angle decayed in time, for a certain range of Mach numbers, the lift coefficient did not decay to zero lift, thereby indicating the presence of nonunique solutions.

In the present study, the same test cases were carried out with the more accurate full potential unsteady code. These computations were carried out in the hope that the root cause of the nonuniqueness appearing from the small disturbance model, might become apparent. It was found that the lift coefficient does decay to zero as the pitch angle decays to zero (Fig. 1). All solutions converged to zero lift with the unsteady full potential code. If the cause of the nonuniqueness is the isentropic flow assumption in potential flow models, then the full potential flow model should also manifest the same property of nonuniqueness.

Considering the array of explanations given for the case of steady flow, a possible explanation for the difference between these small-disturbance and full potential flow results seems to appear from the accurate prediction of downstream pressure. The assumption of a thin body (inherent in the small disturbance code) assumes that the location of the wake to be on a coordinate line downstream of the airfoil. The full potential model, however, moves the wake location with the oscillation airfoil motion. The computational model still involves the assumption of the wake location to be on a coordinate line downstream of the airfoil. However, in this case, this coordinate line is in the computational domain. The transformation to Cartesian coordinates positions the wake on the trailing-edge bisector for symmetric airfoils. This difference in wake position location between small disturbance, and full potential flow models, may account for the difference in the presence of nonunique solutions by providing different down-

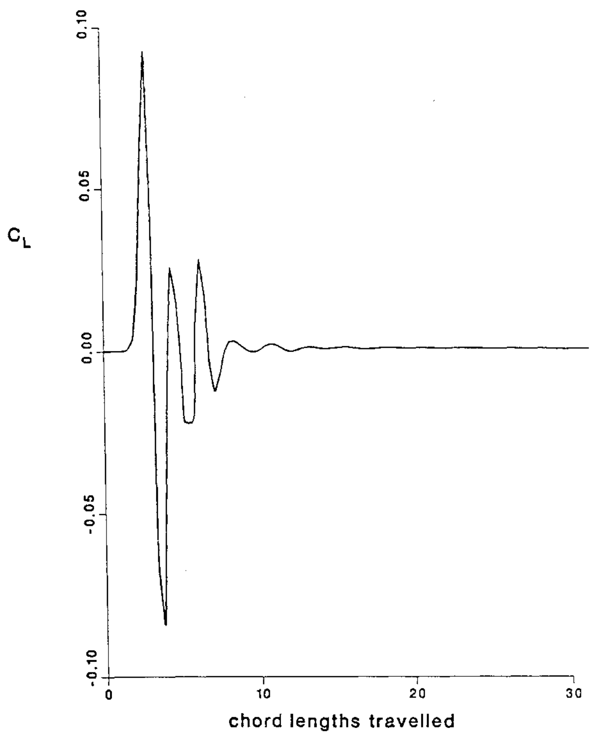

Fig. 1 **Lift coefficient vs time for a NACA 0012 airfoil undergoing a pulse in pitch angle using full potential.**

stream conditions on the upper and lower wake surfaces, that consequently affect flow on the airfoil surfaces.

Unsteady Oscillations

Kerlick and Nixon[8] first pointed out that finite difference, time marching numerical models of the unsteady transonic flow equations, require many cycles of unsteady oscillations in order to reach a quasisteady solution. In addition, it was pointed out that this is observed for a narrow range of Mach numbers between 0.88–0.9. Their studies were carried out with the small disturbance code LTRAN2. A test case for which solutions of lift coefficient with time were given at the following conditions:

$$\text{NACA } 64A006\ M_\infty = 0.9,\ k_c = 0.2,\ \alpha_{pi} = 0.5 \text{ deg } \sin(\omega t)$$

It was found that more than 40 cycles were required for the mean lift of this symmetric airfoil to decay to essentially zero.

In a subsequent article on the same topic, Dowell, Ueda, and Goorjian[9] obtained similar results. These authors asked "Is this nonzero average lift is an artifact of LTRAN2 or do other transonic codes exhibit similar behavior?"

In this study, an answer to this question can be given from the results predicted here. Using the unsteady full potential code developed in this study, the calculated loads were different from those of Kerlick and Nixon. Figure 2 gives the variation-of-lift coefficient C_L with amplitude of unsteady pitch motion. These load variations vary slightly in amplitude for each cycle, indicating that many cycles of unsteady motion are required for a stable oscillatory response of the flowfield. However, it should be noted that the mean lift and moment are zero for each cycle, indicating a difference in the type of results obtained here and those obtained from LTRAN2. Therefore, the answer to the question posed by Dowell et al.[6] is that the nonuniqueness is an artifact of LTRAN2. As in the previous example, the results of the full potential code and small disturbance potential code are different. Hence, the idea that nonuniqueness stems from the lack of accurate entropy modeling is incorrect.

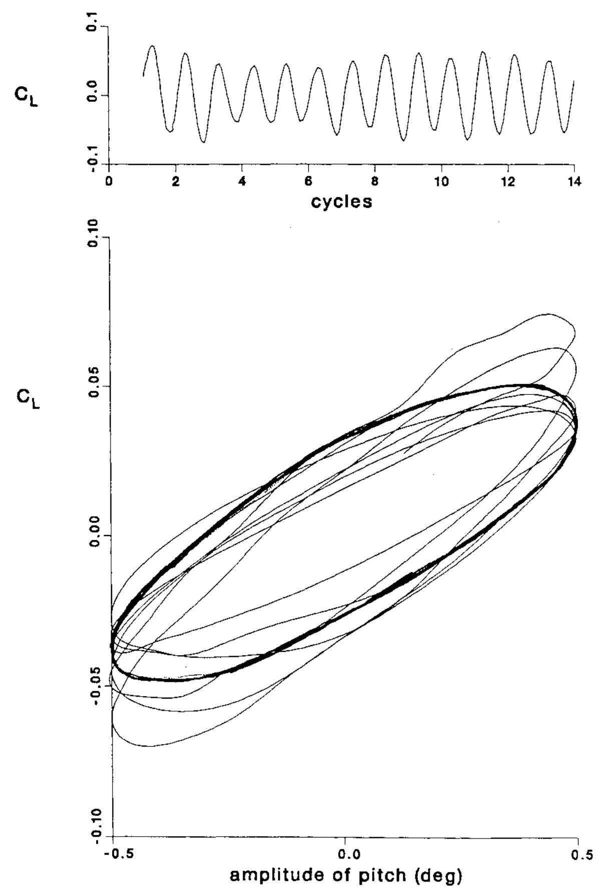

Fig. 2 **Lift coefficient vs angle of attack for a NACA 64A006 airfoil undergoing harmonic pitch motion using full potential.**

Conclusions

The existence of nonunique solutions in unsteady flow has been studied using the full potential flow model. Previous studies provided examples of nonunique solutions using the small disturbance potential flow model; our full potential flow model could not verify those solutions. Therefore, it is not the neglect of shock wave generated vorticity and entropy that is responsible for the nonunique phenomena, because all potential flow models share the assumption of isentropic flow. It is possible that the variation in modeling of the wake boundary conditions may affect the solution obtained. An investigation into the role of the implementation of this boundary condition should be carried out.

Acknowledgments

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Passive Control of Delta Wing Rock

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Introduction

THIS study reports preliminary results on passive control of delta wing rock obtained by adding extended winglets near the leading edge of the wing. The results suggest that the angle of attack at which wing rock occurs can be substantially increased with geometrically simple alternations.

Slender wing rock is a classical problem in aerodynamics; delta wings can provide high-lift forces at relatively large angles of attack. However, their ubiquitous use in high-performance aircraft has been limited by still-existing uncertainties related to their unsteady aerodynamic characteristics,[1] which involve complex, intrinsically three-dimensional separated flows.

One of the first quantitative and analytical studies of the unsteady aerodynamics of delta wing flows at high angles of attacks is that of Ericsson and Reding[2]; using Polhamus[3] leading suction analogy, they concluded that potential-flow theory could be effectively modified to predict adequate results for static loads and that certain unsteady aerodynamics characteristics could be obtained by analyzing the effects of leading-edge vortices. Nguyen et al.[4] focused on the self-induced delta wing rock, a roll oscillation that occurs "naturally" at high angles of attack; with their laboratory results they obtained the now-classic stability boundary relating the limiting angle of attack at which wing rock occurs to the apex angle of the wing. Ericsson[5] confirmed previous observations[4,6] about the two causative separation-induced instabilities; he concluded that large-amplitude delta wing rock is caused by asymmetric leading-edge vortices, while vortex breakdown has a damping effect. He also presented preliminary analytical results for the predictions of wing rock. Recently, Ericsson[7,8] has presented an enhanced comprehensive analytical approach verifying his earlier conclusions.

Studies of passive control of delta wing rock appear to have been very limited; Katz and Levin[9] showed that a delta wing/canard configuration enhanced the roll oscillation at conditions where the unmodified wing would have been stable. In this study we will report results and describe other modifications, one of which actually enhances the wing-rock stability envelope of certain delta wings.

Experimental Equipment

The baseline delta wing model we used in this study was a triangular-shaped plane surface with an apex half-angle of 10.1 deg. (This is the quantity usually referred to as θ_A.) The wing thickness was uniform and equal to 0.5 cm; its chord length 23.28 cm and the length of the base of the delta wing was 8.29 cm. The tests were conducted in the USC low-speed wind tunnel with maximum freestream velocities of 23 m/s. Most tests were conducted at a speed of 12 m/s. The test section is 30 × 30 × 90 cm. The wing was mounted on its base through a shaft with an ultralow-friction bearing on a rotary variable differential transformer which measured the amplitude of the oscillation. The geometric arrangement is identical to that used by Levin and Katz,[6] except that the shaft from the RVDT did not extend over any part of the wing surface. Wing rock was diagnosed both from the RVDT output and by visual observations. The baseline wing was used to screen various modifications; the procedure involved increasing the angle of attack until wing rock was initiated. The angle of attack was then further increased to investigate the stability characteristics. After the optimal modification was determined for the baseline wing, wings with different apex angles and identical modifications were tested to determine the stability boundary.

Laboratory Observations

Several modifications of the baseline delta wing were implemented with the objective to interfere with the vortex formation at the leading edge. The results of these preliminary tests are summarized in Table 1, which compares the critical angle of attack that initiated rock for the different configurations with the critical angle of the baseline wing. The baseline wing is shown in Fig. 1a. In one series of tests (cases 2 and 3), rods of 0.15 cm diam were mounted at the apex at different angles, as shown in Fig. 1b, and then perpendicular to the wing surface at the bottom (case 4), as shown in Fig. 1c. In these tests the motivation was to attempt to produce more symmetric vortex shedding. In another series of tests, a fin was added in the back of the wing as shown in Fig. 1d (cases 5 and 6). Then a series of tests were conducted, where winglets of different sizes and apex-angles were positioned at different distances on either side of the apex of the wing, as shown in Fig. 2 (cases 7–15). These tests indicated that certain sizes of extended winglets resulted in an enhanced envelope of operation. More comprehensive tests were then undertaken to determine the optimal configuration.

Figure 3 shows the stability boundary for the baseline delta wing and the delta wing with extended winglets with a winglet-length to chord-length ratio of 0.43. The ordinate is the angle of attack at which wing rock first appears. The abscissa is the apex half-angle. The figure depicts the curve given in Nguyen et al.[4] (open circles) and the data for the baseline wing obtained in this study (open squares); a comparison of the two curves provides a measure of the errors associated with the laboratory method. The figure also shows the data for the modified wing (solid squares); it is clear that the envelop of operation of the wing is substantially enhanced in the small apex half-angle range. For $\theta_A > 20$ deg the data describes the first occurrence of vortex breakdown. For $\theta_A < 5$ deg, there was no detectable wing rock; the modified wing was unconditionally stable. It is interesting to note that the modified wing curve indicates that similar regimes exist for the modified and unmodified wings.

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